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ON COMPRESSION AND RAREFACTION WAVES IN A DISPERSED LAYER

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Mathematical modeling of nonstationary filtration regimes with instantaneous reduction (increase) in the pressure at entry into the granular bed has been carried out within the framework of the two-temperature approximation. The regularities of transmission of rarefaction and compression waves have been investigated. Equations for calculation of the minimum and maximum temperature of the heat-transfer agent in criterial form and of the time of reaching a new stationary state have been obtained.

Introduction. Nonstationary processes in infiltrated disperse systems, which are caused by sharp changes in the inlet pressure, may occur in the operation of industrial apparatuses in different transient regimes and in off-optimum situations and emergencies. Therefore, modeling of such processes is of undeniable practical interest. This is particularly true of the heat-releasing granular bed in which failures in filtration of the heat-transfer agent may produce uncontrolled changes in its temperature.

The transmission of a rarefaction wave appearing on sharp throwoff in the pressure at entry into the bed by a porous heat-releasing element has been modeled in [1–3]. The investigation was limited to a throwoff of 0.25 atm in the pressure and accordingly small velocities of the heat-carrier gas, which allows us to use Darcy's law. The present work seeks to study the basic regularities of transmission of compression and rarefaction waves by dispersed layers of spherical particles in a fairly wide range of variation in the determining parameters: the layer height, the particle diameter, and the value of the throwoff (rise) in the pressure at entry into the layers.

Formulation of the Problem. We consider a one-dimensional two-temperature model of ideal-gas flow through a disperse medium of spherical particles. The resistance force is calculated from the well-known Ergun formula [4]. The system of equations describing such a process has the following form:

$$\rho_f (1 + \kappa (1 - \varepsilon)) \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) = - \frac{\partial p}{\partial x} - 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{\mu_f u}{d^2} - 1.75 \, \text{sign}(v) \frac{1 - \varepsilon}{\varepsilon^3} \frac{\rho_f u^2}{d} + \tilde{\mu}_f \frac{\partial^2 v}{\partial x^2} , \quad (1)$$

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial}{\partial x} (\rho_f v) = 0 , \quad (2)$$

$$p = \rho_f R T_f , \quad (3)$$

$$\rho_f \varepsilon c_p \left(\frac{\partial T_f}{\partial t} + v \frac{\partial T_f}{\partial x} \right) = \varepsilon \left(\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial x} \left(\varepsilon \lambda_f \frac{\partial T_f}{\partial x} \right) +$$

$$+ \frac{6 (1 - \varepsilon) \alpha}{d} (T_s - T_f) + 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{\mu_f u^2}{d^2} + 1.75 \frac{1 - \varepsilon}{\varepsilon^3} \frac{\rho_f |u|^3}{d} , \quad (4)$$

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$$\rho_s (1-\varepsilon) c_s \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x}\left((1-\varepsilon)\lambda_s \frac{\partial T_s}{\partial x}\right) + \frac{6(1-\varepsilon)\alpha}{d}(T_f - T_s) . \quad (5)$$

System (1)–(5) has been solved with the following boundary conditions:
the initial conditions

$$p(0,x) = p^0(x) , \quad T_f(0,x) = T_f^0(x) , \quad T_s(0,x) = T_s^0(x) , \quad v(0,x) = v^0(x) , \quad \rho_f(0,x) = \rho_f^0(x) , \quad (6)$$

where $p^0(x)$, $T_f^0(x)$, $T_s^0(x)$, $v^0(x)$, and $\rho_f^0(x)$ are the known functions determined by solution of the corresponding stationary problem at the pressure at entry into the layer $p^0(0) = p_0^0$ (atmospheric pressure was prescribed at exit from the bed in all cases);

the boundary conditions

$$x = 0 , \quad p = p_0 , \quad \frac{\partial v}{\partial x} = 0 , \quad \frac{\partial T_f}{\partial x} = 0 \quad \text{for} \quad v < 0 ,$$

$$v_0 \rho_{f0} c_p T_0 = \varepsilon v \rho_f c_p T_f - \varepsilon \lambda_f \frac{\partial T_f}{\partial x} - (1-\varepsilon)\lambda_s \frac{\partial T_s}{\partial x} \quad \text{for} \quad v > 0$$

is the generalized Danckwerts condition [5];

$$(1-\varepsilon)\lambda_s \frac{\partial T_s}{\partial x} = \alpha_0 (T_s - T_0)$$

is the "preheating" of the gas;

$$x = h , \quad p = p_{atm} , \quad \frac{\partial v}{\partial x} = 0 , \quad \frac{\partial T_f}{\partial x} = \frac{\partial T_s}{\partial x} = 0 \quad [5] . \quad (7)$$

The condition $\frac{\partial v}{\partial x} = 0$ for $x = 0$ and $x = h$ has been taken in accordance with the recommendations of [6]. It is necessary for solving the problem for the gas flow rate $J_f(t, x)$ not known in advance and determined in the process of solution. The term with an effective viscosity $\tilde{\mu}_f$, which makes the order of the equation higher, has been introduced in a standard manner into (1).

Reduction to a Dimensionless Form. The procedure of making system (1)–(7) dimensionless is nontrivial because of the arbitrariness in selection of the time and gas-velocity scales. These scales are established in the following manner. We write Eq. (1) in dimensionless form without specifying t^* and v^*:

$$\frac{\rho_{f0} h}{p_{atm}} (1 + \kappa(1-\varepsilon)) \rho_f' \left(\frac{v^*}{t^*}\frac{\partial v'}{\partial t'} + \frac{(v^*)^2}{h} v' \frac{\partial v'}{\partial \xi}\right) = -\frac{p_0 - p_{atm}}{p_{atm}} \frac{\partial p'}{\partial \xi}$$

$$- 150 \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 \frac{\mu_f v^* h}{d^2 p_{atm}} v' - 1.75 \,\text{sign}(v') \frac{1-\varepsilon}{\varepsilon} \frac{\rho_{f0} (v^*)^2 h}{d p_{atm}} \rho_f' (v')^2 + \frac{\tilde{\mu}_f v^*}{h p_{atm}} \frac{\partial^2 v'}{\partial \xi^2} . \quad (8)$$

It is necessary that the left-hand side of (8) has the simplest form

$$(1 + \kappa(1-\varepsilon)) \rho_f' \left(\frac{\partial v'}{\partial t'} + v' \frac{\partial v'}{\partial \xi}\right) .$$

This yields two conditions

$$\frac{\rho_{f0} h v^*}{p_{atm} t^*} = 1 \ , \quad \frac{\rho_{f0} (v^*)^2}{p_{atm}} = 1 \ , \tag{9}$$

enabling us to determine t^* and v^* as follows:

$$t^* = \sqrt{\frac{\rho_{f0}}{p_{atm}}}\, h = \frac{h}{\sqrt{RT_0}} \ , \quad v^* = \sqrt{\frac{p_{atm}}{\rho_{f0}}} = \sqrt{RT_0} \ . \tag{10}$$

As is seen, the characteristic velocity determined by (10) is equal, accurate to the factor $\sqrt{\gamma}$, to the velocity of sound in the gas v_{sd} at the temperature T_0 and atmospheric pressure. Then, with account for (10), we write system (1)–(7) in dimensionless form as

$$\rho'_f (1 + \kappa (1 - \varepsilon)) \left(\frac{\partial v'}{\partial t'} + v' \frac{\partial v'}{\partial \xi} \right) = - \frac{p_0 - p_{atm}}{p_{atm}} \frac{\partial p'}{\partial \xi}$$

$$- 150 \left(\frac{1 - \varepsilon}{\varepsilon} \right)^2 \frac{\mu_f}{\mu_{f0}} \frac{1}{\mathrm{Re}} \left(\frac{h}{d} \right)^2 v' - 1.75 \ \mathrm{sign}\ (v') \frac{1 - \varepsilon}{\varepsilon} \frac{h}{d} \rho'_f (v')^2 + \frac{\tilde{\mu}_f}{\mu_{f0}} \frac{1}{\mathrm{Re}} \frac{\partial^2 v'}{\partial \xi^2} \ , \tag{11}$$

$$\frac{\partial \rho'_f}{\partial t'} + \frac{\partial}{\partial \xi} (\rho'_f v') = 0 \ , \tag{12}$$

$$\rho'_f = \left(p' \left(\frac{p_0}{p_{atm}} - 1 \right) + 1 \right) \Big/ (\theta_f + 1) \ , \tag{13}$$

$$\rho'_f \varepsilon \left(\frac{\partial \theta_f}{\partial t'} + v' \frac{\partial \theta_f}{\partial \xi} \right) = \varepsilon P \left(\frac{\partial p'}{\partial t'} + v' \frac{\partial p'}{\partial \xi} \right) + \frac{\partial}{\partial \xi} \left(\varepsilon \frac{1}{\mathrm{Pe}_f} \frac{\partial \theta_f}{\partial \xi} \right)$$

$$+ \frac{1}{\mathrm{Pe}^f} (\theta_s - \theta_f) + 150 \frac{(1 - \varepsilon)^2}{\varepsilon} \frac{\mu_f}{\mu_{f0}} \frac{1}{\mathrm{Re}} \frac{h}{d} \tilde{P} (v')^2 + 1.75 (1 - \varepsilon) \tilde{P} \rho'_f \left| v' \right|^3 \ , \tag{14}$$

$$(1 - \varepsilon) \frac{\partial \theta_s}{\partial t'} = \frac{\partial}{\partial \xi} \left((1 - \varepsilon) \frac{1}{\mathrm{Pe}_s} \frac{\partial \theta_s}{\partial \xi} \right) + \frac{1}{\mathrm{Pe}^s} (\theta_f - \theta_s) \tag{15}$$

with the initial conditions

$$p' (0, \xi) = \frac{p^0 (x) - p_{atm}}{p_0 - p_{atm}} \ , \quad \theta_f (0, x) = \frac{T_f^0 (x) - T_0}{T_0} \ , \quad \theta_s (0, \xi) = \frac{T_s^0 (x) - T_0}{T_0} \ ,$$

$$v' (0, \xi) = \frac{v^0 (x)}{v^*} \ , \quad \rho'_f (0, \xi) = \frac{\rho_f^0 (x)}{\rho_{f0}} \tag{16}$$

and the boundary conditions

$$\xi = 0\,,\quad p' = 1\,,\quad \frac{\partial v'}{\partial \xi} = 0\,,\quad \frac{\partial \theta_f}{\partial \xi} = 0 \quad \text{for} \quad v' < 0\,;$$

$$\frac{\partial \theta_f}{\partial \xi} = \mathrm{Pe}_f \rho_f' v' \theta_f - \frac{(1-\varepsilon)}{\varepsilon} \frac{\lambda_s}{\lambda_f} \frac{\partial \theta_s}{\partial \xi} \quad \text{for} \quad v' > 0\,;$$

$$\frac{\partial \theta_s}{\partial \xi} = \frac{\alpha_0 h}{(1-\varepsilon)\,\lambda_s}\,\theta_s\,;\qquad \xi = 1\,,\quad p' = 0\,,\quad \frac{\partial v'}{\partial \xi} = 0\,,\quad \frac{\partial \theta_f}{\partial \xi} = \frac{\partial \theta_s}{\partial \xi} = 0\,. \tag{17}$$

Parameters of the Theoretical Model. The thermal conductivities of the gas and the ensemble of particles have been calculated from the formulas [7]

$$\lambda_f = \lambda_f^m + 0.5 \rho_f c_p \frac{u}{\varepsilon} d\,, \tag{18}$$

$$\lambda_s = \lambda_r + \frac{\lambda_c - \varepsilon \lambda_f^m}{1-\varepsilon}\,, \tag{19}$$

where

$$\lambda_c = \lambda_f^m \left(\frac{\lambda_s^m}{\lambda_f^m} \right)^{(1-\varepsilon)\left(\frac{\lambda_s^m}{\lambda_f^m} \right)^{-0.06}}\,;\quad \lambda_r = \frac{16}{3} \left(0.35 + 0.52 \varepsilon_s^{0.85} \right) \sigma T_s^3 d\,.$$

The heat-exchange coefficients α and α_0 have been calculated from the dependences known from the literature and given respectively in [8] and [9]. In the calculations, we took air as the gas and silica as particles.

Analysis of Results. Figure 1 shows the characteristic profiles of p, v, ρ_f, J_f, and T_f at different instants of time for the rarefaction wave. The initial and final stationary distributions of these quantities are given here. The emerginq return flow is characterized by the sharp increase in the velocity on the inlet portion. Also, we observe a considerable cooling of the heat-transfer agent behind the wave front, which is due to the conversion of the internal energy of the gas to a kinetic one. To calculate the minimum gas temperature, we obtain the following interpolation dependence:

$$\theta_{\min} = -\,1.9 \left(\frac{d}{h} \right)^{0.20} \left(\frac{p_0^0 - p_0}{p_0 - p_{\mathrm{atm}}} \right)^{0.12} \left(\frac{p_0^0 - p_0}{T_0 \rho_{f0} c_p} \right)^{0.24} \mathrm{Re}^{-0.10}\,, \tag{20}$$

which reflects the influence of the controlling factors. The time of reaching a new stationary state can be evaluated from the dependence

$$\frac{\Delta t}{t^*} = 1.37 \cdot 10^6\, \mathrm{Re}^{-0.61} \left(\frac{p_0^0 - p_0}{p_0 - p_{\mathrm{atm}}} \right)^{0.18}\,. \tag{21}$$

Figure 2 gives the characteristic profiles of p, v, ρ_f, J_f, and T_f at different instants of time for the compression wave. As is seen, the behavior of the curves in this case and in Fig. 1 is different. A characteristic feature of the compression wave is sharp increase in the velocity and temperature of the gas behind its front. For the maximum gas temperature to be calculated we have obtained the following interpolation dependence:

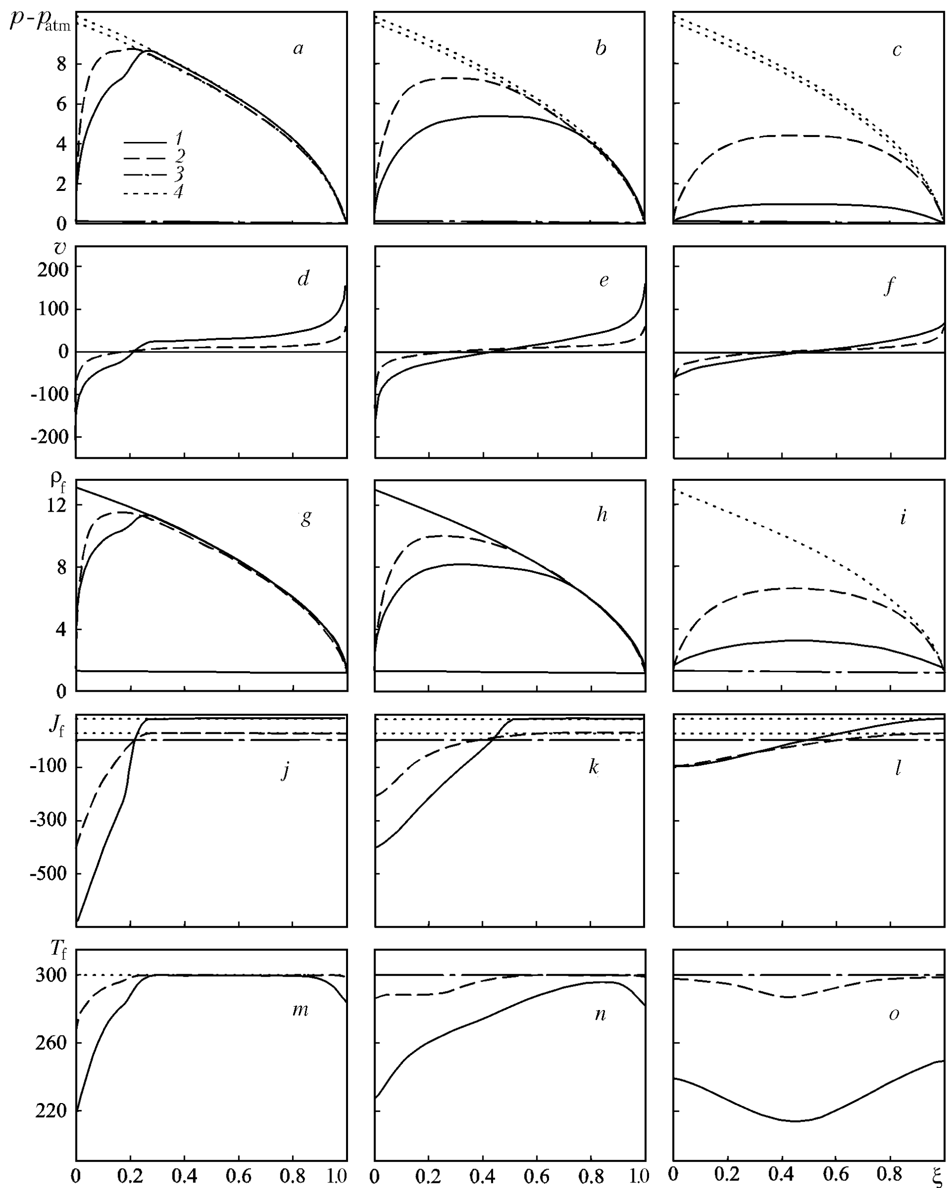

Fig. 1. Pressure (a, b, and c), velocity (d, e, and f), density (g, h, and i), flow-rate (j, k, and l), and temperature (m, n, and o) profiles of the gas for the rarefaction wave at different instants of time for p_0^0 = 11 atm and 1.1 atm; a, d, g, j, and m) t = 0.002, b, e, h, k, and n) 0.0082, and c, f, i, l, and o) 0.041 sec; 1) d = 0.4 and 2) 0.04 m; 3) final states; 4) initial ones (the upper curve, d = 0.04 m, the lower curve, 0.4 m). p, atm; v, m/sec; ρ_f, kg/m^3; J_f, kg/(m^2·sec); T_f, K.

$$\theta_{max} = 0.78 \left(\frac{d}{h}\right)^{0.11} \left(\frac{p_0 - p_0^0}{p_0 - p_{atm}}\right)^{0.16} \left(\frac{p_0 - p_0^0}{T_0 \rho_{f0} c_p}\right)^{0.37} \mathrm{Re}^{-0.074} . \quad (22)$$

The correlation

$$\frac{\Delta t}{t^*} = 5.7 \left(\frac{d}{h}\right)^{-0.40} \left(\frac{p_0 - p_0^0}{T_0 \rho_{f0} c_p}\right)^{0.70} \quad (23)$$

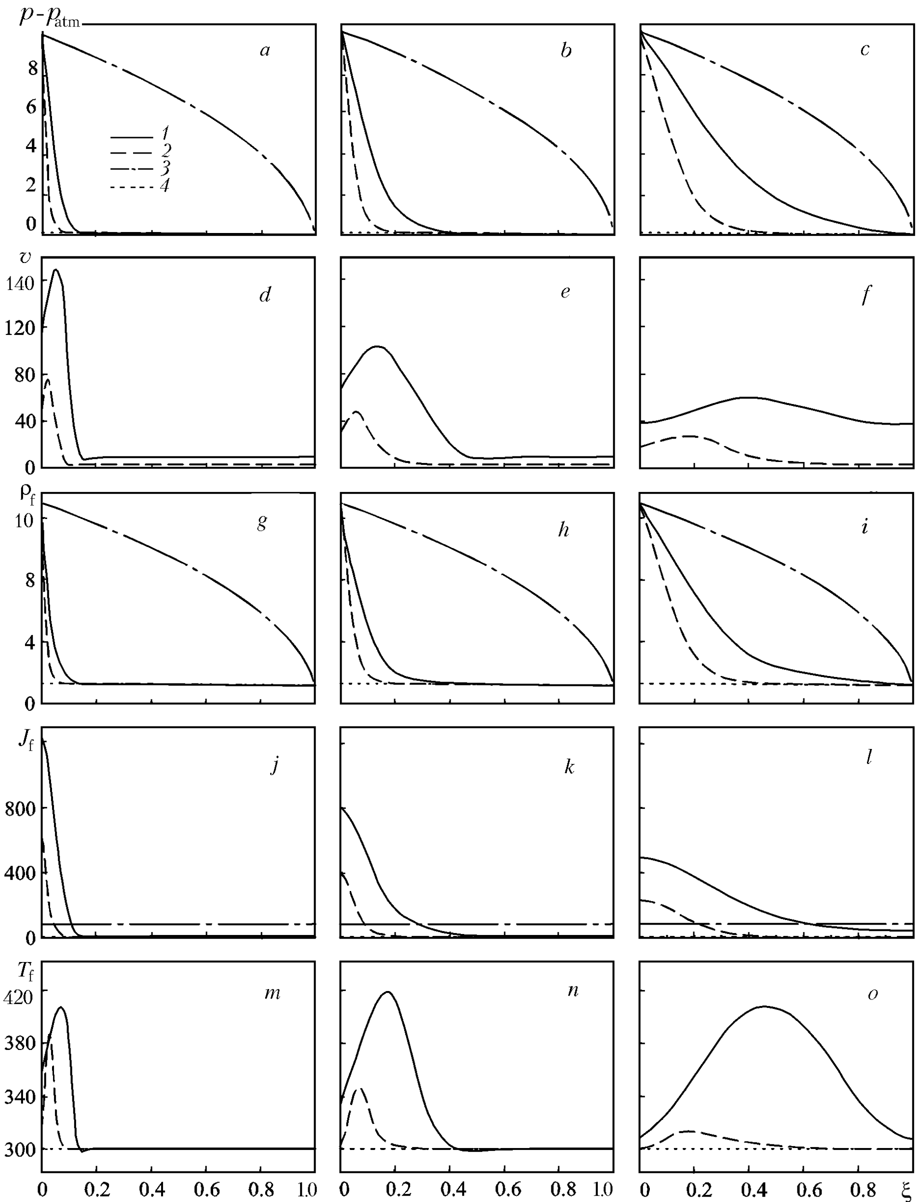

Fig. 2. Pressure (a, b, and c), velocity (d, e, and f), density (g, h, and i), flowrate (j, k, and l), and temperature (m, n, and o) profiles of the gas for the compression wave at different instants of time for p_0^0 = 1.1 atm and p_0 = 11 atm. Notation a–o is the same as in Fig. 1. p, atm; v, m/sec; ρ_f, kg/m^3; J_f, kg/(m^2·sec); T_f, K.

has been obtained for the time of reaching a new stationary state.

Since the entire process of reaching the new stationary state was very short and its duration did not exceed 1 sec in the investigated range of variation in the quantities p_0^0 and p_0 (for the compression wave, $1.1 \le p_0^0 \le 3$ atm and $3 \le p_0 \le 11$ atm; for the rarefaction wave, $3 \le p_0^0 \le 11$ atm and $1.1 \le p_0 \le 3$ atm), the particle temperature remained constant, in practice.

Figure 3 shows results of calculation of the wave-front velocity from the formula

$$v_w = \frac{x_w}{t_w}, \tag{24}$$

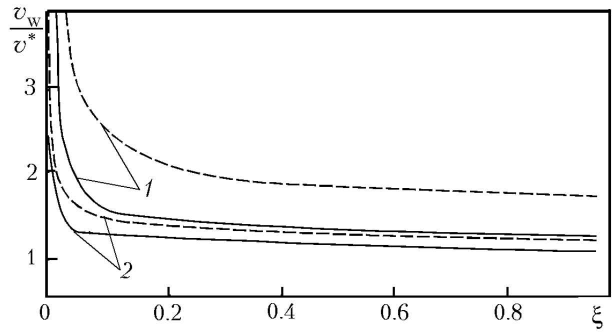

Fig. 3. Wave-front velocity vs. dimensionless coordinate: solid curve, rarefaction wave; dashed curve, compression wave; 1) d = 0.4 and 2) 0.04 m.

where x_w is the minimum value of the coordinate x at the instant of time t_w for which the running value $p(t, x)$ differs from the initial value $p^0(t, x)$ by no more than 1%. As is seen, the wave velocities were dependentt on jus the particle diameter.

Conclusions. The numerical investigation carried out has shown that the character of the compression and rarefaction waves formed in the disperse medium is substantially dependent on the structural parameters of the fill and the value of the initial jump in the pressure at entry into the system. The influence of particles on the character of gasdynamic processes in the gas–solid two-phase system is allowed for by the simplex d/h which represents the relation of the micro- and macroscale of the system (see (20), (22), and (23)). It is the most pronounced in the dependence for the time of reaching a new stationary state for the compression wave (23). At the same time, the analogous dependence for the rarefaction wave (21) is self-similar in d/h.

The scale factor (layer height) exerts the greatest influence on the time of reaching a new stationary state: $\Delta t \sim h^{0.39}$ (rarefaction wave) and $\Delta t \sim h^{1.4}$ (compression wave). At the same time, the values of the maximum and minimum temperatures are dependent on h weaker: $\theta_{\min} \sim h^{-0.3}$ and $\theta_{\max} \sim h^{-0.17}$.

NOTATION

c_p and c_V, specific heat of the gas at constant pressure and at constant volume respectively, J/(kg·K); c_s, specific heat of particles, J/(kg·K); d, diameter of particles, m; h, height of the granular bed, m; $J_f = \rho_f \varepsilon v_f$, mass flow rate of the gas, kg/(m^2·sec); p, pressure, Pa; $p' = \dfrac{p - p_{atm}}{p_0 - p_{atm}}$; $P = \dfrac{p_0 - p_{atm}}{T_0 \rho_{f0} c_p}$; $\tilde{P} = \dfrac{p_{atm}}{T_0 \rho_{f0} c_p} \dfrac{h}{d}$; $\text{Pe}^f = \dfrac{c_p \rho_{f0} v^*}{6(1-\varepsilon)\alpha} \dfrac{d}{h}$, $\text{Pe}^s = \dfrac{c_s \rho_s v^*}{6(1-\varepsilon)\alpha} \dfrac{d}{h}$, $\text{Pe}_f = \dfrac{c_p \rho_{f0} h v^*}{\lambda_f}$, and $\text{Pe}_s = \dfrac{c_s \rho_s h v^*}{\lambda_s}$, Péclet numbers; $\text{Re} = \dfrac{\rho_{f0} h v^*}{\mu_{f0}}$, Reynolds number; R, gas constant (for air), m^2/(sec^2·K); t, time, sec; $t' = t/t^*$; T_f and T_s, temperatures of the gas and particles respectively, K; T_0, inlet temperature of the gas, K; u, filtration velocity of the gas, m/sec; v, gas velocity in the gaps between particles ($v = u/\varepsilon$), m/sec; v_w, wave velocity, m/sec; $v_{sd} = \sqrt{\dfrac{\gamma p_{atm}}{\rho_{f0}}}$, velocity of sound at the temperature T_0 and atmospheric pressure, m/sec; v_0, inlet velocity of the gas, m/sec; $v' = v/v^*$; x, coordinate, m; α and α_0, heat-exchange coefficients, W/(m^2·K); $\gamma = c_p/c_V$, adiabatic exponent; ε, porosity; ε_s, emissivity factor of particles; $\theta_f = (T_f - T_0)/T_0$; $\theta_s = (T_s - T_0)/T_0$; κ, factor of apparent mass ($\kappa = 0.5$); λ_f and λ_s, thermal conductivity of the gas and the ensemble of particles, W/(m·K); λ_s^m, thermal conductivity of the particle material, W/(m·K); μ_f, dynamic viscosity of the gas, kg/(m·sec); μ_{f0}, dynamic viscosity of the gas at the temperature T_0, kg/(m·sec); $\xi = x/h$; ρ_f and ρ_s, density of the gas and particles, kg/m^3; ρ_{f0}, density of the gas at the temperature T_0 and pressure p_{atm}, kg/m^3; $\rho' = \rho_f/\rho_{f0}$;

σ, Stefan–Boltzmann constant, $W/(m^2 \cdot K^4)$. Subscripts and superscripts: f, gas; m, molecular; s, particles; 0, initial stationary state (superscript) and inlet state (subscript); atm, atmospheric; c, conductive; r, radiative; sd, sound; w, wave; min, minimum; max, maximum.

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